THE ELLIPTICAL DIELECTRIC WAVEGUIDE
A VERY USEFUL MODEL FOR SOME MICROWAVE AND MILLIMETER WAVE
INTEGRATED CIRCUITS AND COMPONENTS

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ABSTRACT

Guided modes spectrum of the elliptical dielectric waveguide has been completed and correspondence between modes in circular, elliptical, rectangular and slab dielectric waveguides have been investigated with details. Based on the results, an elliptical model description technique for planar waveguides has been suggested. This technique has, by exemple, been shown to be applicable for lower order modes in microslot lines and open suspended strip-line. Elliptical dielectric waveguide is also shown as a very practical structure to approximate dielectric resonator of high permittivity with various shapes.

I - INTRODUCTION

The guiding properties of some important microwave and millimeter wave integrated circuit have been considered in a great number of recent papers. All they show the extreme mathematical complêxity which is encountered to obtain rigourously dispersion characteristics of guided modes in planar structures. In contrast, when the mathematical approach is very simplified it is only an approximate technique available on some limited cases.

Nowaday, in spite of their multiplicity efficiency of some planar waveguides to be used as transmission lines remains a very important problem. This is due to the lack of informations about practical parameters such as like - T.E.M. characteristic impedances, dielectric and "skin" losses coefficients. These last practical data are estimated from knowledge of mode field components which are not always easily deduced from the specific numerical approach of the dispersion properties.

This paper shows how, from an approximate elliptical model description of some well known planar structure, the classical waveguide treatment yields both dispersion characteristics and fields components of fundamental and some higher order modes.

II - THE ELLIPTICAL DIELECTRIC WAVEGUIDE

The first of the main ideas of the study described in this paper is that the elliptical dielectric waveguide can approximate a great number of different shapes. So, depending upon the eccentricity e such as :

$$e = \begin{cases} 1/\cosh \xi_o \\ 1/\{1+(2/W_D)^2\}^{1/2} \end{cases}$$

the elliptical rod can take the form of a circular waveguide ($e \sim 0$), an approximate rectangular waveguide ($.9 < e < .1$) and at last a slab dielectric waveguide ($e \sim 1$) as shown in figure 1.

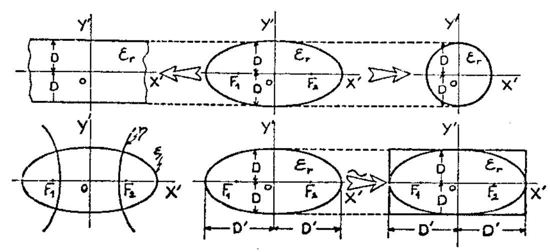

FIG. 1

Fortunately, an exact analytic solution, although very involved, exists for the guided mode spectrum of the elliptical dielectric waveguide. However, only detailed theoretical as well as experimental results have been given only on dominant modes e HE_{11} and o EH_{11} by C.YEH[1]. Thus, it was of our interest to complete them by propagation properties of higher order modes.

The appropriate solution of the waveguide treatment are of two types because the symmetry mirror about the plane y'=0. So, even and odd modes are respectively defined by :

$$\text{even(e)} \begin{cases} Ez(x',-y') = Ez(x',y') \\ \vec{E}_T(x',-y') = -\vec{E}_T(x',y') \\ Hz(x',-y') = -Hz(x',y') \\ \vec{H}_T(x',-y') = \vec{H}_T(x',y') \end{cases} \qquad \text{odd(o)} \begin{cases} Ez(x',-y') = -Ez(x',y') \\ \vec{E}_T(x',-y') = \vec{E}_T(x',y') \\ Hz(x',-y') = Hz(x',y') \\ \vec{H}_T(x',-y') = -\vec{H}_T(x',y') \end{cases}$$

where subscripts z and T denote longitudinal and transverse field components. Since on cylindrical structures (ie circular, elliptical, rectangular...) symmetries are principally governed by azimuthal functions, longitudinal components Ez and Hz can be expressed by :

$$Hz_1 = \sum_n A_n \begin{cases} se_n(\eta,\gamma_1^2) \,.\, Se_n(\xi,\gamma_1^2) & (e) \\ ce_n(\eta,\gamma_1^2) \,.\, Ce_n(\xi,\gamma_1^2) & (o) \end{cases}$$
$$Ez_1 = \sum_n B_n \begin{cases} ce_n(\eta,\gamma_1^2) \,.\, Ce_n(\xi,\gamma_1^2) & (e) \\ se_n(\eta,\gamma_1^2) \,.\, Se_n(\xi,\gamma_1^2) & (o) \end{cases} \qquad (1)$$

in region (1): $0 \leq \xi \leq \xi_o$; $0 \leq \eta \leq 2\Pi$. For region (o) : $\xi_o \leq \xi < \infty$; $0 \leq \eta \leq 2\Pi$, these components are defined as :

$$Hz_o = \sum_n L_n \begin{cases} se_n(\eta,-\gamma_o^2).Gek_n(\xi,-\gamma_o^2) & (e) \\ ce_n(\eta,-\gamma_o^2).Fek_n(\xi,-\gamma_o^2) & (o) \end{cases}$$
$$Ez_o = \sum_n P_n \begin{cases} ce_n(\eta,-\gamma_o^2).Fek_n(\xi,-\gamma_o^2) & (e) \\ se_n(\eta,-\gamma_o^2).Gek_n(\xi,-\gamma_o^2) & (o) \end{cases} \qquad (2)$$

where γ_1^2 and $-\gamma_o^2$ represent respectively $(k_1^2 - \beta^2)q^2/4$ and $(k_o^2 - \beta^2)q^2/4$ with $k_1^2 = \omega^2\mu_o\varepsilon_o\varepsilon_r$ and $k_o^2 = \omega^2\mu_o\varepsilon_o$. β is the phase constant of the waves. q is the half focal distance. A_n, B_n, L_n and P_n are arbitrary constants related by boundary conditions. Azimuthal and radial functions are periodic Mathieu functions[2] with integer order n because the periodicity of Π or 2Π exists for the waves. A modes classifying system in the elliptical dielectric waveguide can then, be given in relation with the mode classifying system proposed by E. SNITZER[3] in the circular dielectric waveguide. So, for a mode m, determinantal equations, derived from boundary conditions get value of the ratio of arbitrary constant A_m and B_m as :

$$\alpha_{e \text{ or } o} = \frac{\omega\mu_o}{\beta}\frac{A_m}{B_m}$$

ω : pulsation of the harmonic wave
β : phase constant of the harmonic wave

This dimensionless ratio informs about the TE or TM charactor of the hybrid mode m. The hybrid modes are designated by following their α value in the "far from cutoff" situation. So, the modes classifying system is :

$0<\alpha_e<-1 \longrightarrow$ e HE_{mp} $\quad 0<\alpha_o<1 \longrightarrow$ o EH_{mp}

$1<\alpha_e<\infty \longrightarrow$ e EH_{mp} $\quad -\infty<\alpha_o<-1 \longrightarrow$ o HE_{mp}

where p denote the p^{th} parametric zero of characteristic equation of the mode m.

III - ELLIPTICAL MODELS FOR PLANAR STRUCTURES

Previous results allow to follow variation of the dispersion characteristics and field components of the guided modes when the elliptical cross section is changed gradually in some way between slab (W/D $\sim$3 or e $\sim$.9) and circular (W/D $\sim$.1 or e $\sim$.1) shapes. So according their symmetries properties about the plane y'=0 agreement between corresponding modes in various structures are summarized in Table I.

DIELECTRIC WAVEGUIDES	SLAB	ELLIPTICAL	CIRCULAR
Fundamental modes (no cut-off)	oTM_0 eTE_0	o EH_{11} e HE_{11}	e,o HE_{11}
higher order modes $q=0,1$	eTM_{2q+1} oTE_{2q+1}	e $EH_{0,q+1}$ o $HE_{0,q+1}$	e $TM_{0,q+1}$ o $TE_{0,q+1}$
$q=1,2$	oTM_{2q} eTE_{2q}	o $EH_{1,q+1}$ e $HE_{1,q+1}$	e,o $HE_{1,q+1}$

TABLE I

In the elliptical model description for planar structures it will be always assumed that most of the power flow is confined inside the new structures so as to make their elliptical geometry no longer important as far as dispersion characteristics and fields components of modes are concerned. Moreover, as between slab and elliptical dielectric rod it will be always supposed that guided modes in planar structures are connected to some guided modes of the elliptical models in a quasi similar manner as described in Table I. Neththeless some precautions are necessary when planar structures and models are in fact waveguides with baffles as it will be illustrated in following section on microslot lines[4] (see fig. 2).

IV - GUIDED MODES SPECTRUM OF SOME PLANAR WAVEGUIDES

a) Symmetrical slot-line (figure 2.a)

Actual and model symmetrical slot-line are waveguides with baffles because propagating modes in it, must satisfy the boundary conditions :

$$Ez = 0 \text{ and } E_\xi = 0 \text{ for } \eta = 0 \text{ and } \eta = \Pi$$

They take the place of the periodicity for the waves in elliptical dielectric waveguide without baffles. Appropriate solutions for axial components Ez and Hz must be choosen among azimuthal and radial Mathieu functions with both rational and integer order n but only the second type of solutions have been considered in this work. Modes in symmetrical slot-line may be constructed as following :

. *odd modes* : these modes are characterized by the absence of the ξ components of the electric field at the planes of the baffles so, they are identical to odd modes in elliptical dielectric waveguide without baffles.

. *even modes* : they are obtained from relations(1) and (2) for even waves by substitutions :

$$(3)\quad \begin{aligned} se_n(\eta,\gamma_1^2) &\longrightarrow \pm ce_n(\eta,\gamma_1^2) \\ ce_n(\eta,\gamma_1^2) &\longrightarrow \pm se_n(\eta,\gamma_1^2) \\ se_n(\eta,-\gamma_o^2) &\longrightarrow \pm ce_n(\eta,\gamma_o^2) \\ ce_n(\eta,-\gamma_o^2) &\longrightarrow \pm se_n(\eta,\gamma_o^2) \end{aligned} \quad \text{and} \quad \begin{aligned} Se_n(\xi,\gamma_1^2) &\rightarrow Fe_n(\xi,\gamma_1^2) \\ Ce_n(\xi,\gamma_1^2) &\rightarrow Ge_n(\xi,\gamma_1^2) \\ Gek_n(\xi,-\gamma_o^2) &\rightarrow Fek_n(\xi,-\gamma_o^2) \\ Fek_n(\xi,-\gamma_o^2) &\rightarrow Gek_n(\xi,-\gamma_o^2) \end{aligned}$$

with sign + if $\pi < \eta < 0$ and sign - if $\pi < \eta < 2\pi$

Radial functions Fe_n and Ge_n are defined as :

$$Fe_n(o,\gamma_1^2) = 0 \quad ; \quad Ge'_n(o,\gamma_1^2) \qquad (4)$$

Such symmetries choosen in Hz and Ez expressions and relations(4) garanty the continuity of axial components and their first derivatives across the slot plane ($|x'| < q$; $y' = 0$). Components Hz suffers a discontinuity at $\eta = 0$ and π for $\xi > 0$, due to currents flowing on each side of the baffles conducting surfaces. The electric field at the plane of the slot is tangential to the slot and acquires infinite values at the points of the baffles.

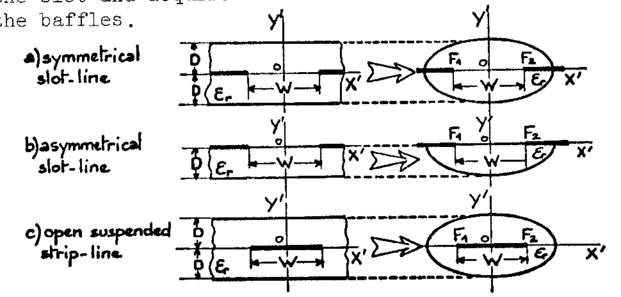

FIG. 2

b) Asymmetrical slot-line (figure 2.b)

Typical symmetries related above do not exist in this case. Thus,modes in asymmetrical slot-line model can be constructed as linear combinations of previous modal solutions on symmetrical slot-line.

c) Open suspended strip-line (figure 2.c)

Periodicity of Π or 2Π for the waves appears again on this structure and appropriate solutions are constructed using Mathieu function with integer order only. Modes are of two types :

. *odd modes* : which are similar to odd modes in symmetrical slot-line and odd modes in elliptical dielectric waveguide without strip.

. *even modes*: substitutions described above by relations (3) are operated once again but in expressions (1) and (2) for odd waves.

V - VARIOUS RESULTS ON PRACTICAL PARAMETERS

In symmetrical slot-line model, two zero cut-off modes are found to be propagated : the slot mode refferred to as the fundamental baffled mode e HE_{01} and the classical o EH_{11} surface mode of the elliptical rod not perturbed by conducting baffles. Modes in asymmetrical slot-line are all baffled modes and the fundamental one, e HE_{01} has also a zero cut-off frequency. At last in open suspended strip-line only the o EH_{11} surface mode has a zero cut-off frequency. Figures 3 give some results on lowest order modes in various structures.

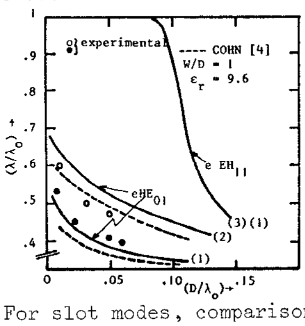

FIG. 3
No cut-off modes of :
1) symmetrical slot-line
2) asymmetrical slot-line
3) open-suspended strip-line.

For slot modes, comparisons with "second order" COHN's results, together with experimental measurements are also given in figure 3. All measurements are in good agreement except ones on symmetrical slot-line. Indeed, in its realization an incontrolable air-film between substrates especially at the neighbourhood of the slot reduces widely the relative permittivity (of superimposed alumina substrates). Dispersion measurements are thus, very critical on this structure.

Besides, efficiency of a structure described above to be used as a transmission line can be examine since, field components are straightforwardly derived from inspection of modes functions. So, like-TEM impedances for any modes $(\vec{E}_m, \vec{H}_m)$ can be defined as :

$$Z_{CI} = \frac{2p}{II^*} \text{ with } I = \int_{L'} \vec{H}_m . \vec{dl}$$

where L' is a portion of the "peripherical" curve C yielding direct current.
or as :

$$Z_{CV} = \frac{VV^*}{2p} \text{ with } V = \int_A^B \vec{E}_m . \vec{dl}$$

where V is the maximum voltage between two any points A and B in waveguide cross section.
or at last as :

$$Z_{CVI} = \frac{V}{I}$$

In Z_{CI}, Z_{CV} expressions above, p is the power flow of the mode m. Figure 4 shows these parameters for fundamental baffled modes in microslot lines. COHN's results are also drawn in this figure allowing a direct comparison.

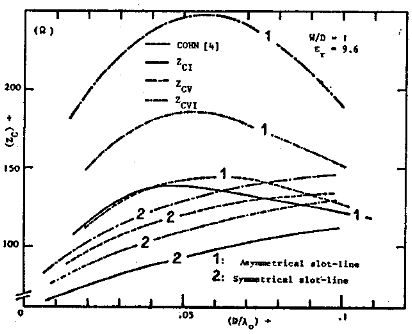

FIG. 4

Like-T.E.M. characteristic impedances of slot-lines.

Attenuation coefficient can also be calculated from this new approximate treatment by classical formulas. It is :

$$\alpha_D = 8.686 \frac{\sigma_D}{4p} \int \vec{E}_m . \vec{E}_m^* dS \quad \text{(dB/meters)}$$

for dielectric losses. For "skin" losses it is :

$$\alpha_C = 8.686 \frac{\omega\mu_0\delta}{8p} \oint_C \vec{H}_m . \vec{H}_m^* dl \quad \text{(dB/meters)}$$

σ_D is the dielectric conductivity. S is the cross sectional dielectric medium. δ is the "skin" depth. C is a closed "peripherical" curve inside conductors. Figure 5 give only some theoretical estimations of "skin" losses for fundamental baffled modes in microslot lines. No correction term taking into account metallization process and surface roughness has been used in "skin" depth expression.

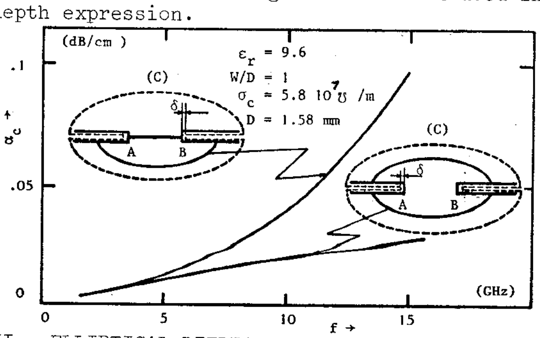

FIG. 5

VI - ELLIPTICAL DIELECTRIC RESONATOR

Natural resonance frequency of the fundamental magnetic dipole mode can be estimated from the well known *first order* and *second order* approximations[5]. However, it is possible to improve the previous approximations by a *third order* which is particularly convenient for study of composite resonator together with the important temperature frequency stabilisation problem.[6]

Geometrical dimensions of the elliptical dielectric resonator are thickness H, major axis 2a, minor axis 2b, focal distance 2q and finally the eccentricity e.

In this *third order*, electric and magnetic fields of the fundamental magnetic dipole mode satisfy radiation condition by starting from all the walls. So, dispersion characteristics of o TE_{01} and o HE_{01} modes propagated by respectively the *closed* and the *opened* elliptical dielectric waveguide are identified at any point (ω, β). This numerical comparison yields on equivalent *closed* waveguide $(a_{eq}, e, \varepsilon_{r\ eq})$ which take the place of the *opened* waveguide (a, e, ε_r) at any given frequency. Then at any frequency, the second order approximation is applied to find resonance condition of the resonator $(a_{eq}, e, \varepsilon_{r\ eq}, H)$.

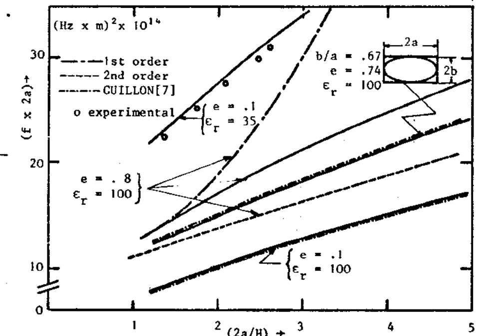

FIG. 6

Resonator characteristics.

The figure 6 shows the resonance frequency of a resonator (2a, e = 0.8, ε_r = 100, H). The *first* and *second* order approximation are also plotted in this figure. When the eccentricity becomes zero (in practice e ∿ .1), a cylindrical resonator (D = 2a, ε_r, H) is obtained. On this particular shape, the figure 6 set off excellent agreement between this *third order* approximation and results published by Y. GARAULT and P. GUILLON[7]. Besides, this *third order* gives a good interpretation of the phenomenon since theoretical and experimental values agree within 1 %. Figure 6 describes a very important property of the elliptical dielectric resonator. Indeed, resonance frequency of a rectangular resonator (2a, 2b, ε_r, H) is quasi identical to resonance frequency of an elliptical dielectric resonator (2a, e, ε_r, H) whose cross-section is drawn into the rectangle (2a, 2b). Eccentricity of the elliptical line is then given by $e = (1 - b^2/a^2)^{1/2}$.

VII - CONCLUSION

An elliptical model description for some planar waveguides has been made. At present time all experimental results have been always found in a correct order of magnitude with theoretical predictions. If more accurate calculations are required on actual structures, corrections can be made using well-known perturbation method by regarding actual structure to be an elliptical one with addition of dielectric material where it is necessary.

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